Responsive Choice in Mobile Processes

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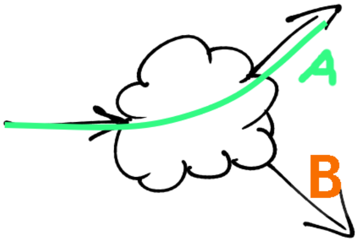
[1] Joint work with António Ravara

Choice

Choice

Definition (Selection $A \vee B$)

I will either behave like A or like B



Definition (Branching $A + B$)

You can make me do A or B

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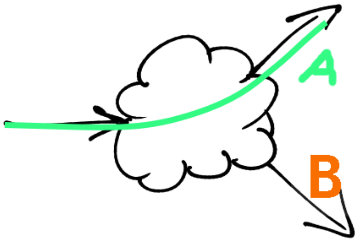

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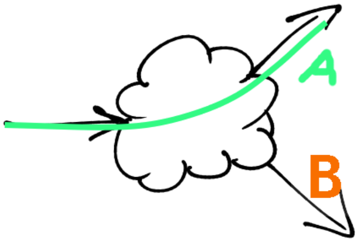


Definition (Branching $A + B$)

You can make me do A or B

The Synchronous Polyadic π-calculus

$$!\, n(xy).(\nu tf)(\bar{a}\langle tf\rangle \mid (t.\bar{y} + f.\bar{x}))$$

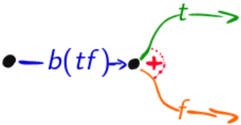
Input
Binding
Output
Sum
Replication
Sequence
Parallel Composition

Choice Examples (I)

- Data Encodings

$$b := \text{True} \quad \stackrel{\text{def}}{=} \quad !\, b(tf).\overline{t}$$

$$b := \text{False} \quad \stackrel{\text{def}}{=} \quad !\, b(tf).\overline{f}$$

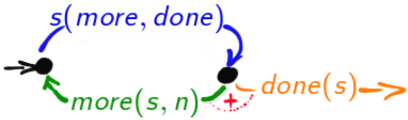


$$\text{If } b \text{ Then } P \text{ Else } Q \quad \stackrel{\text{def}}{=} \quad \overline{b}(\nu tf).(t.P+f.Q)$$

Choice Examples (II)

- Client-Server Conversations

$$\overline{prod}(\boldsymbol{\nu} s).s(more, done).\overline{more}(\boldsymbol{\nu} s, 2).$$
$$s(more, done).\overline{more}(\boldsymbol{\nu} s, 5).$$
$$s(more, done).\overline{done}(\boldsymbol{\nu} s).s(x).\overline{print}\langle x\rangle$$



$$\mathbf{!}\, prod(s).\overline{p_0}\langle 1, s\rangle \quad | \quad \mathbf{!}\, p_0(t, s).\overline{s}(\boldsymbol{\nu} more, done).$$
$$\big(more(s, n).\overline{p_0}\langle t \times n, s\rangle + done(s).\overline{s}\langle r\rangle\big)$$

Liveness

Liveness Properties

Definition (Activeness $p_{\mathbf{A}}$)

I am soon ready to receive (send) at p

$$\overline{print}_{\mathbf{A}} \models \overline{prod}(\nu s).s(more, done).\overline{more}(\nu s, 2).$$
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$$s(more, done).\overline{done}(\nu s).s(x).\overline{print}\langle x\rangle \quad |$$
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& \mathbf{!}\, prod(s).\overline{p_0}\langle 1, s\rangle \quad | \quad \mathbf{!}\, p_0(t, s).\overline{s}(\boldsymbol{\nu} more, done).\\
& \big(more(s, n).\overline{p_0}\langle t \times n, s\rangle + done(s).\overline{s}\langle r\rangle\big)
\end{aligned}
$$

Liveness Properties

Definition (Responsiveness $p_{\mathbf{R}}$)

If I get (send) a message from (to) *you* at p, I'll obey p's protocol

$$\overline{prod}_{\mathbf{R}} \quad \not\models \quad \overline{prod}(\nu s).s(more, done).\overline{more}(\nu s, 2).$$
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Channel Types

... I'll obey p's "protocol"?

Definition (Channel Type a.k.a Protocol)

$$(\text{parameters} \quad ; \quad \text{input} \quad ; \quad \text{output})$$

- Bool $\stackrel{\text{def}}{=}$ $((;;)(;;) \quad ; \quad \bar{1}_{\mathbf{A}} \vee \bar{2}_{\mathbf{A}} \quad ; \quad 1_{\mathbf{A}} + 2_{\mathbf{A}})$
- $!\, rand(xy).(\nu q)\,(\bar{q}|q.\bar{x}|q.\bar{y}) \quad | \quad \overline{rand}\langle tf\rangle\,|\,(t.P+f.Q)$
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- $\mathbf{!}\, n(xy).(\boldsymbol{\nu} tf)\,(\bar{a}\langle tf\rangle \mid (t.\bar{y} + f.\bar{x}))$

Behavioural Statements

Behavioural Statements

Say something about processes' behaviour:

Definition (Behavioural Statements Δ, Ξ, ...)

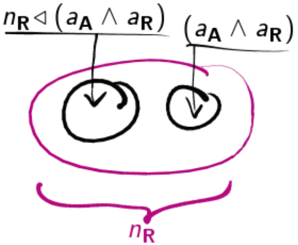
$$\Delta \;::=\; \Delta \vee \Delta \;\mid\; \Delta + \Delta \;\mid\; \Delta \wedge \Delta \;\mid\; \Delta \lhd \Delta \;\mid\; p_{\mathbf{A}} \;\mid\; p_{\mathbf{R}} \;\mid\; \cdots$$

Causality

Definition (Dependency $A \triangleleft B$)

If you give me B, I'll give you A.

$$n_{\mathbf{R}} \triangleleft (a_{\mathbf{A}} \wedge a_{\mathbf{R}}) \models \,!\, n(xy).\overline{a}(\boldsymbol{\nu} tf).(t.\overline{y} + f.\overline{x})$$



Semantics, Example

$$\left(n_{\mathbf{R}} \triangleleft a_{\mathbf{AR}} \quad ; \quad !\, n(xy).\overline{a}(\nu tf).(t.\bar{y}+f.\bar{x})\right)$$

$$\downarrow n(t'f')$$

$$\left((\bar{t}'_{\mathbf{A}} \vee \bar{f}'_{\mathbf{A}}) \triangleleft a_{\mathbf{AR}} \quad ; \quad \cdots \mid \overline{a}(\nu tf).(t.\bar{f}'+f.\bar{t}')\right)$$

$$\downarrow \overline{a}(\nu tf)$$

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$$\downarrow t \qquad\qquad\qquad\qquad \downarrow f$$

$$\left((\bar{t}'_{\mathbf{A}} \vee \bar{f}'_{\mathbf{A}}) \quad ; \quad \cdots \mid (\bar{f}')\right) \checkmark \qquad \left((\bar{t}'_{\mathbf{A}} \vee \bar{f}'_{\mathbf{A}}) \quad ; \quad \cdots \mid (\bar{t}')\right) \checkmark$$

Semantics, Example

$$\left(n_{\mathbf{R}} \lhd a_{\mathbf{AR}} \quad ; \quad \mathbf{!}\, n(xy).\overline{a}(\nu tf).(t.\overline{y} + f.\overline{x})\right)$$

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Semantics, Example

$$\left(n_{\mathbf{R}} \lhd a_{\mathbf{AR}} \;;\; \mathbf{!}\,n(xy).\overline{a}(\boldsymbol{\nu} tf).(t.\overline{y}+f.\overline{x})\right)$$

$$\Big\downarrow\, n(t'f')$$

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Type System

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Our type system:

- Takes: A process P, and channel types Σ

$$P : \quad !\, n(xy).(\nu tf)\,(\overline{a}\langle tf\rangle \mid (t.\bar{y} + f.\bar{x}))$$

$$\Sigma : \quad \{a : \mathrm{Bool}, n : \mathrm{Bool}\}$$

- Produces: A *correct* logical formula Ξ describing P

$$\Xi : \quad n_{\mathbf{R}} \triangleleft a_{\mathbf{AR}}$$

- is Decidable

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Typing — Composition

$$(\bar{t}_{\mathbf{A}} \vee \bar{f}_{\mathbf{A}}) \triangleleft a_{\mathbf{AR}} \vdash \bar{a}\langle tf \rangle$$

$$((t_{\mathbf{A}} \wedge \bar{y}_{\mathbf{A}} \triangleleft \bar{t}_{\mathbf{A}}) + (f_{\mathbf{A}} \wedge \bar{x}_{\mathbf{A}} \triangleleft \bar{f}_{\mathbf{A}})) \vdash (t.\bar{y} + f.\bar{x})$$

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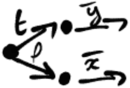

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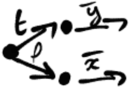

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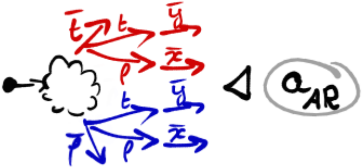
Typing — ∨-Distributivity

∨-Distributivity

"∨ distributes with everything else"

$$(\overline{t}_{\mathbf{A}} \vee \overline{f}_{\mathbf{A}}) \triangleleft a_{\mathbf{AR}} \wedge ((t_{\mathbf{A}} \wedge \overline{y}_{\mathbf{A}} \triangleleft \overline{t}_{\mathbf{A}}) + (f_{\mathbf{A}} \wedge \overline{x}_{\mathbf{A}} \triangleleft \overline{f}_{\mathbf{A}})) =$$

$$(\overline{t}_{\mathbf{A}} \triangleleft a_{\mathbf{AR}} \wedge ((t_{\mathbf{A}} \wedge \overline{y}_{\mathbf{A}} \triangleleft \overline{t}_{\mathbf{A}}) + (f_{\mathbf{A}} \wedge \overline{x}_{\mathbf{A}} \triangleleft \overline{f}_{\mathbf{A}}))) \quad \vee$$
$$(\overline{f}_{\mathbf{A}} \triangleleft a_{\mathbf{AR}} \wedge ((t_{\mathbf{A}} \wedge \overline{y}_{\mathbf{A}} \triangleleft \overline{t}_{\mathbf{A}}) + (f_{\mathbf{A}} \wedge \overline{x}_{\mathbf{A}} \triangleleft \overline{f}_{\mathbf{A}})))$$



Typing — Reduction (I)

Branching Reduction

$$(\bar{p}_{\mathbf{A}} \wedge (p_{\mathbf{A}} + \varepsilon)) \hookrightarrow (\bar{p}_{\mathbf{A}} \wedge p_{\mathbf{A}}) \vee (\bar{p}_{\mathbf{A}} \wedge (p_{\mathbf{A}} + \varepsilon))$$

$$(\bar{t}_{\mathbf{A}} \triangleleft a_{\mathbf{AR}} \wedge ((t_{\mathbf{A}} \wedge \bar{y}_{\mathbf{A}} \triangleleft \bar{t}_{\mathbf{A}}) + (f_{\mathbf{A}} \wedge \bar{x}_{\mathbf{A}} \triangleleft \bar{f}_{\mathbf{A}}))) \quad \vee$$

$$(\bar{f}_{\mathbf{A}} \triangleleft a_{\mathbf{AR}} \wedge ((t_{\mathbf{A}} \wedge \bar{y}_{\mathbf{A}} \triangleleft \bar{t}_{\mathbf{A}}) + (f_{\mathbf{A}} \wedge \bar{x}_{\mathbf{A}} \triangleleft \bar{f}_{\mathbf{A}}))) \hookrightarrow$$

$$(\bar{t}_{\mathbf{A}} \wedge t_{\mathbf{A}} \wedge (\bar{y}_{\mathbf{A}} \triangleleft \bar{t}_{\mathbf{A}})) \triangleleft a_{\mathbf{AR}} \quad \vee \quad (\bar{f}_{\mathbf{A}} \wedge f_{\mathbf{A}} \wedge (\bar{x}_{\mathbf{A}} \triangleleft \bar{f}_{\mathbf{A}})) \triangleleft a_{\mathbf{AR}}$$

Typing — Reduction (I)

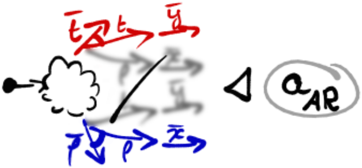
Branching Reduction

$$(\bar{p}_{\mathbf{A}} \wedge (p_{\mathbf{A}} + \varepsilon)) \hookrightarrow (\bar{p}_{\mathbf{A}} \wedge p_{\mathbf{A}}) \vee (\bar{p}_{\mathbf{A}} \wedge (p_{\mathbf{A}} + \varepsilon))$$

$$(\bar{t}_{\mathbf{A}} \triangleleft a_{\mathbf{AR}} \wedge ((t_{\mathbf{A}} \wedge \bar{y}_{\mathbf{A}} \triangleleft \bar{t}_{\mathbf{A}}) + \cancel{(f_{\mathbf{A}} \wedge \bar{x}_{\mathbf{A}} \triangleleft \bar{f}_{\mathbf{A}})})) \quad \vee$$

$$(\bar{f}_{\mathbf{A}} \triangleleft a_{\mathbf{AR}} \wedge (\cancel{(t_{\mathbf{A}} \wedge \bar{y}_{\mathbf{A}} \triangleleft \bar{t}_{\mathbf{A}})} + (f_{\mathbf{A}} \wedge \bar{x}_{\mathbf{A}} \triangleleft \bar{f}_{\mathbf{A}}))) \hookrightarrow$$

$$(\bar{t}_{\mathbf{A}} \wedge t_{\mathbf{A}} \wedge (\bar{y}_{\mathbf{A}} \triangleleft \bar{t}_{\mathbf{A}})) \triangleleft a_{\mathbf{AR}} \quad \vee \quad (\bar{f}_{\mathbf{A}} \wedge f_{\mathbf{A}} \wedge (\bar{x}_{\mathbf{A}} \triangleleft \bar{f}_{\mathbf{A}})) \triangleleft a_{\mathbf{AR}}$$



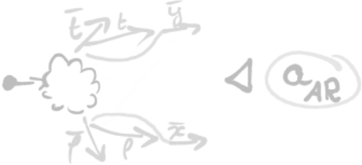
Typing — Reduction (II)

Dependency Reduction

$$(\alpha \lhd \beta) \wedge (\beta \lhd \gamma) \quad \hookrightarrow \quad (\alpha \lhd \beta \vee \gamma) \wedge (\beta \lhd \gamma)$$

$$((t_A \wedge \bar{t}_A) \lhd a_{AR} \wedge (\bar{y}_A \lhd \bar{t}_A)) \quad \vee \quad ((f_A \wedge \bar{f}_A) \lhd a_{AR} \wedge (\bar{x}_A \lhd \bar{f}_A)) \hookrightarrow$$

$$((t_A \wedge \bar{t}_A \wedge \bar{y}_A) \lhd a_{AR}) \quad \vee \quad ((f_A \wedge \bar{x}_A \wedge \bar{f}_A) \lhd a_{AR}) =$$



$$((t_A \wedge \bar{t}_A \wedge \bar{y}_A) \quad \vee \quad (f_A \wedge \bar{f}_A \wedge \bar{x}_A)) \lhd \; a_{AR} \vdash \; \bar{a}\langle tf\rangle \mid (t.\bar{y} + f.\bar{x})$$

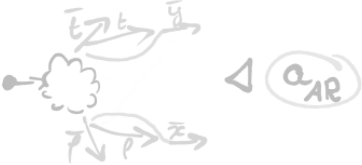
Typing — Reduction (II)

Dependency Reduction

$$(\alpha \lhd \beta) \wedge (\beta \lhd \gamma) \quad \hookrightarrow \quad (\alpha \lhd \beta \vee \gamma) \wedge (\beta \lhd \gamma)$$

$$((t_{\mathbf{A}} \wedge \bar{t}_{\mathbf{A}}) \lhd a_{\mathbf{AR}} \wedge (\bar{y}_{\mathbf{A}} \lhd \bar{t}_{\mathbf{A}})) \quad \vee \quad ((f_{\mathbf{A}} \wedge \bar{f}_{\mathbf{A}}) \lhd a_{\mathbf{AR}} \wedge (\bar{x}_{\mathbf{A}} \lhd \bar{f}_{\mathbf{A}})) \hookrightarrow$$

$$((t_{\mathbf{A}} \wedge \bar{t}_{\mathbf{A}} \wedge \bar{y}_{\mathbf{A}}) \lhd a_{\mathbf{AR}}) \quad \vee \quad ((f_{\mathbf{A}} \wedge \bar{x}_{\mathbf{A}} \wedge \bar{f}_{\mathbf{A}}) \lhd a_{\mathbf{AR}}) =$$



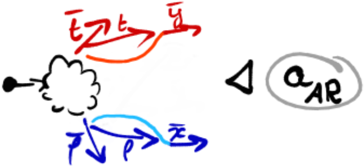
$$((t_{\mathbf{A}} \wedge \bar{t}_{\mathbf{A}} \wedge \bar{y}_{\mathbf{A}}) \quad \vee \quad (f_{\mathbf{A}} \wedge \bar{f}_{\mathbf{A}} \wedge \bar{x}_{\mathbf{A}})) \lhd a_{\mathbf{AR}} \vdash \bar{a}\langle tf\rangle \mid (t.\bar{y} + f.\bar{x})$$

Typing — Reduction (II)

Dependency Reduction

$$(\alpha \lhd \beta) \wedge (\beta \lhd \gamma) \quad \hookrightarrow \quad (\alpha \lhd \beta \vee \gamma) \wedge (\beta \lhd \gamma)$$

$$((t_{\mathbf{A}} \wedge \bar{t}_{\mathbf{A}}) \lhd a_{\mathbf{AR}} \wedge (\bar{y}_{\mathbf{A}} \lhd \bar{t}_{\mathbf{A}})) \quad \vee \quad ((f_{\mathbf{A}} \wedge \bar{f}_{\mathbf{A}}) \lhd a_{\mathbf{AR}} \wedge (\bar{x}_{\mathbf{A}} \lhd \bar{f}_{\mathbf{A}})) \hookrightarrow$$
$$((t_{\mathbf{A}} \wedge \bar{t}_{\mathbf{A}} \wedge \bar{y}_{\mathbf{A}}) \lhd a_{\mathbf{AR}}) \quad \vee \quad ((f_{\mathbf{A}} \wedge \bar{x}_{\mathbf{A}} \wedge \bar{f}_{\mathbf{A}}) \lhd a_{\mathbf{AR}}) =$$



$$((t_{\mathbf{A}} \wedge \bar{t}_{\mathbf{A}} \wedge \bar{y}_{\mathbf{A}}) \quad \vee \quad (f_{\mathbf{A}} \wedge \bar{f}_{\mathbf{A}} \wedge \bar{x}_{\mathbf{A}})) \lhd a_{\mathbf{AR}} \vdash \bar{a}\langle tf \rangle \mid (t.\bar{y} + f.\bar{x})$$

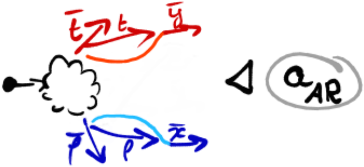
Typing — Reduction (II)

Dependency Reduction

$$(\alpha \triangleleft \beta) \wedge (\beta \triangleleft \gamma) \quad \hookrightarrow \quad (\alpha \triangleleft \beta \vee \gamma) \wedge (\beta \triangleleft \gamma)$$

$$((t_{\mathbf{A}} \wedge \bar{t}_{\mathbf{A}}) \triangleleft a_{\mathbf{AR}} \wedge (\bar{y}_{\mathbf{A}} \triangleleft \bar{t}_{\mathbf{A}})) \quad \vee \quad ((f_{\mathbf{A}} \wedge \bar{f}_{\mathbf{A}}) \triangleleft a_{\mathbf{AR}} \wedge (\bar{x}_{\mathbf{A}} \triangleleft \bar{f}_{\mathbf{A}})) \hookrightarrow$$

$$((t_{\mathbf{A}} \wedge \bar{t}_{\mathbf{A}} \wedge \bar{y}_{\mathbf{A}}) \triangleleft a_{\mathbf{AR}}) \quad \vee \quad ((f_{\mathbf{A}} \wedge \bar{x}_{\mathbf{A}} \wedge \bar{f}_{\mathbf{A}}) \triangleleft a_{\mathbf{AR}}) =$$



$$((t_{\mathbf{A}} \wedge \bar{t}_{\mathbf{A}} \wedge \bar{y}_{\mathbf{A}}) \quad \vee \quad (f_{\mathbf{A}} \wedge \bar{f}_{\mathbf{A}} \wedge \bar{x}_{\mathbf{A}})) \triangleleft a_{\mathbf{AR}} \vdash \bar{a}\langle tf \rangle \mid (t.\bar{y} + f.\bar{x})$$

Typing — Responsiveness

$$n_{\mathbf{R}} \triangleleft \ \ldots \ \vdash \ P = \mathbf{!}\, n(xy).(\boldsymbol{\nu} tf)\,(\bar{a}\langle tf\rangle \mid (t.\bar{y} + f.\bar{x}))$$

Responsiveness

$$(\boldsymbol{\nu}\tilde{y})\,\left(n : \sigma \odot n_{\mathbf{R}} \triangleleft \sigma\{^{\tilde{y}}/_{1\cdots n}\} \odot \cdots\right) \vdash n(\tilde{y})$$

$$(\boldsymbol{\nu} xy)\,\left(n_{\mathbf{R}} \triangleleft (\bar{x}_{\mathbf{A}} \vee \bar{y}_{\mathbf{A}}) \ \wedge \ (\bar{x}_{\mathbf{A}} \vee \bar{y}_{\mathbf{A}}) \triangleleft a_{\mathbf{AR}}\right) \vdash P$$

$$\hookrightarrow n_{\mathbf{R}} \triangleleft a_{\mathbf{AR}} \quad \vdash \quad \mathbf{!}\, n(xy).(\boldsymbol{\nu} tf)\,(\bar{a}\langle tf\rangle \mid (t.\bar{y} + f.\bar{x}))$$

Typing — Responsiveness

$$n_{\mathbf{R}} \triangleleft \ldots \vdash P = \mathbf{!}\, n(xy).(\nu tf)\,(\overline{a}\langle tf\rangle \mid (t.\overline{y} + f.\overline{x}))$$

Responsiveness

$$(\nu\tilde{y})\,\left(n:\sigma \odot n_{\mathbf{R}}\triangleleft \sigma\{^{\tilde{y}}/_{1\ldots n}\} \odot \cdots\right) \vdash n(\tilde{y})$$

$$(\nu xy)\,\left(n_{\mathbf{R}}\triangleleft(\overline{x}_{\mathbf{A}} \vee \overline{y}_{\mathbf{A}}) \wedge (\overline{x}_{\mathbf{A}} \vee \overline{y}_{\mathbf{A}})\triangleleft a_{\mathbf{AR}}\right) \vdash P$$

$$\hookrightarrow n_{\mathbf{R}}\triangleleft a_{\mathbf{AR}} \vdash \mathbf{!}\, n(xy).(\nu tf)\,(\overline{a}\langle tf\rangle \mid (t.\overline{y}+f.\overline{x}))$$

Typing — Responsiveness

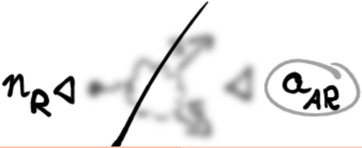
$$n_{\mathbf{R}} \lhd \ldots \vdash P = !\, n(xy).(\boldsymbol{\nu} tf)\,(\bar{a}\langle tf\rangle \mid (t.\bar{y} + f.\bar{x}))$$

Responsiveness

$$(\boldsymbol{\nu}\tilde{y})\left(n : \sigma \odot n_{\mathbf{R}} \lhd \sigma\{^{\tilde{y}}/_{1\cdots n}\} \odot \cdots\right) \vdash n(\tilde{y})$$

$$(\boldsymbol{\nu} xy)\left(n_{\mathbf{R}} \lhd (\bar{x}_{\mathbf{A}} \vee \bar{y}_{\mathbf{A}}) \;\wedge\; (\bar{x}_{\mathbf{A}} \vee \bar{y}_{\mathbf{A}}) \lhd a_{\mathbf{AR}}\right) \vdash P$$

$$\hookrightarrow n_{\mathbf{R}} \lhd a_{\mathbf{AR}} \quad \vdash \quad !\, n(xy).(\boldsymbol{\nu} tf)\,(\bar{a}\langle tf\rangle \mid (t.\bar{y} + f.\bar{x}))$$



Summary

Type Language		Process Behaviour
Selection & Branching	$A \vee B,\ p + q$	Choice
Activeness & Responsiveness	$p_{\mathbf{A}},\ p_{\mathbf{R}}$	Liveness
Dependencies	$\gamma \triangleleft \varepsilon$	Causality

Type System:

- Decidable
- Constructs Logical Formulæ
- Sound
- Compositional

Thank you!

In the paper:

- Encoding and verification of the “and” Boolean operator.
- Safety properties: Multiplicities.
- Description of the Typed Labelled Transition System.
- Detailed survey of related works.
- ...

More info:

- http://maxime.gamboni.org/

Supplementary Material

Safety Properties

Definition (Multiplicity)

p^m: I'll use p *at most* m times.

- p^0: Never
- p^1: Once
- p^ω: Once, replicated
- $p^\star$: Many times

$$p^\omega \wedge \overline{p}^0 \wedge p_0^\omega \wedge \overline{p_0}^\star \models \,!\, p(s).\overline{p_0}(1,s) \quad | \quad !\, p_0(t,s).\overline{s}(\nu\, more, done).\big(more(s,n).\overline{p_0}(t \times n, s) + done(s).\overline{s}\langle r\rangle\big)$$

Behavioural Statement Syntax

- Behavioural statements
  $\Delta \;::=\; \Delta \vee \Delta \;\mid\; \Delta \wedge \Delta \;\mid\; \Delta \triangleleft \Delta \;\mid\; \gamma \;\mid\; p^m \;\mid\; \bot \;\mid\; \top$
- Resources $\gamma \;::=\; s_{\mathbf{A}} \;\mid\; p_{\mathbf{R}}$
- Sums $s \;::=\; s + s \;\mid\; p$

Type System Rules (I)

$$\frac{-}{(\varnothing; \top \blacktriangleleft \top) \vdash \mathbf{0}} \text{ (R-NIL)}$$

$$\frac{\forall i : \Gamma_i \vdash P_i}{\Gamma_1 \odot \Gamma_2 \vdash P_1 \mid P_2} \text{ (R-PAR)} \qquad \frac{\Gamma \vdash P \qquad \Gamma(x) = \sigma}{(\boldsymbol{\nu} x)\,\Gamma \vdash (\boldsymbol{\nu} x : \sigma)\, P} \text{ (R-RES)}$$

$$\frac{\begin{array}{c}\forall i : \big(\mathsf{sub}(G_i) = \{p_i\}, \quad (\Sigma_i; \Xi_{\mathrm{L}i} \blacktriangleleft \Xi_{\mathrm{E}i}) \vdash G_i.P_i\big) \\ \Xi_{\mathrm{E}} \preceq \bigwedge_i \Xi_{\mathrm{E}i} \\ \big(\Xi_{\mathrm{E}} \text{ has concurrent environment } p_{i'}\big) \Rightarrow \varepsilon = \bot\end{array}}{\big(\bigwedge_i \Sigma_i; (\sum_i p_i)_{\mathbf{A}} \triangleleft \varepsilon \wedge \bigvee_i \Xi_{\mathrm{L}i} \blacktriangleleft \Xi_{\mathrm{E}}\big) \vdash \sum_i G_i.P_i} \text{ (R-SUM)}$$

Type System Rules (II)

$$\frac{\begin{array}{c}\Gamma \vdash P \quad \mathsf{sub}(G) = p \quad \mathsf{obj}(G) = \tilde{x} \\ (\#(G) = 1 \text{ and } m' = \star) \Rightarrow \varepsilon = \bot\end{array}}{\begin{array}{r}(\boldsymbol{\nu}\mathsf{bn}(G))\Bigg(\Big(p : \sigma;\ \blacktriangleleft\ p^{m} \wedge \bar{p}^{m'}\Big) \quad \odot \\ \Big(;\ p_{\mathbf{A}}^{\#(G)} \triangleleft \varepsilon \blacktriangleleft\Big) \quad \odot \\ (;\ p_{\mathbf{R}} \triangleleft \sigma[\tilde{x}] \blacktriangleleft) \quad \odot \\ \overline{\sigma}[\tilde{x}] \triangleleft \bar{p}_{\mathbf{AR}} \quad \odot \\ \Gamma \triangleleft \bar{p}_{\mathbf{A}} \quad \Bigg) \vdash G.P\end{array}} \ (\text{R-PRE})$$