# Activeness and Responsiveness in Mobile Processes

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May 27, 2009

[1] Joint work with António Ravara

# This Work

*Characterising* two *liveness properties* in a *mobile process* through the use of a *type system*.

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- Types: descriptions
- Type Semantics: formal meaning
- Type System: computable algorithm

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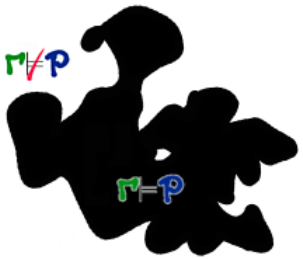


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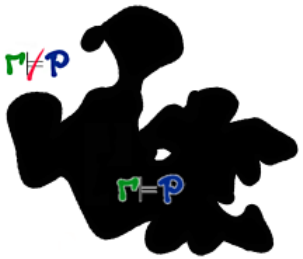


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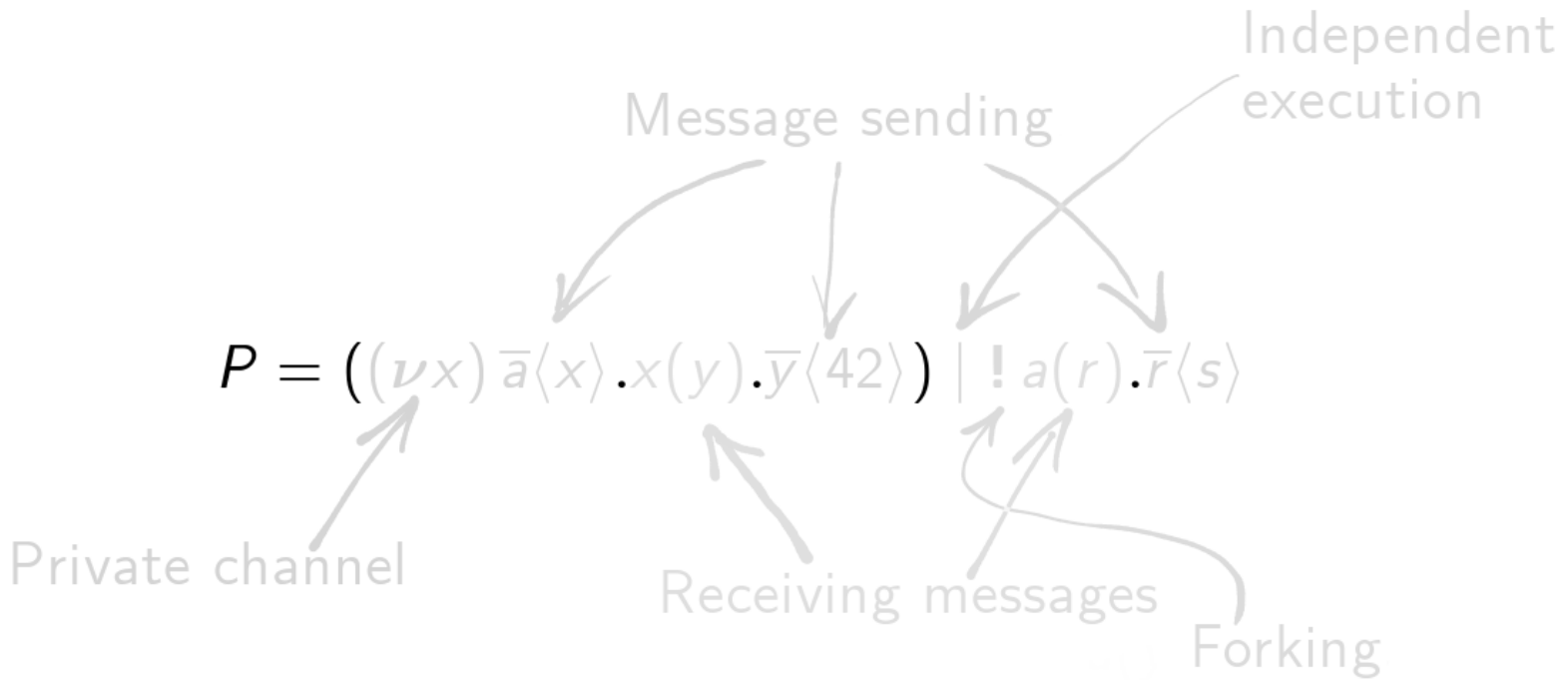

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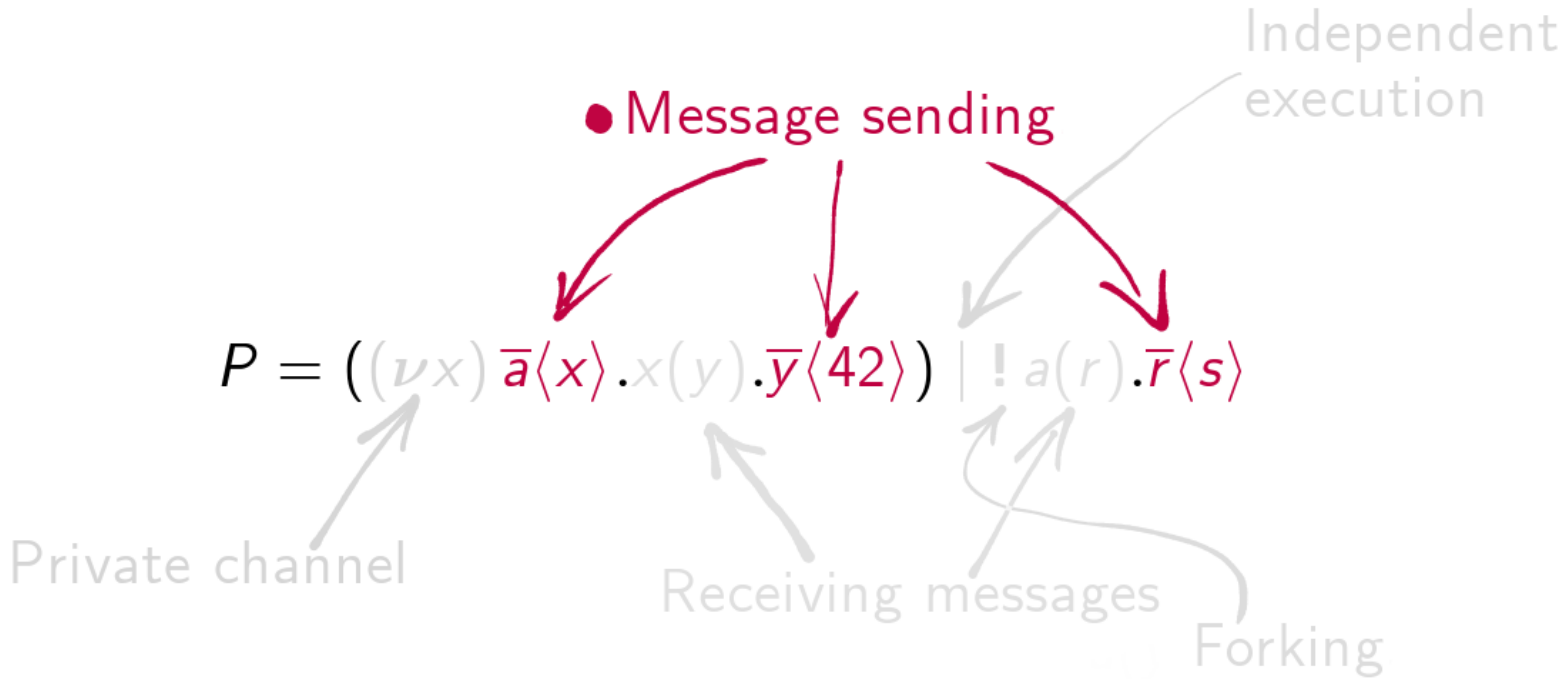

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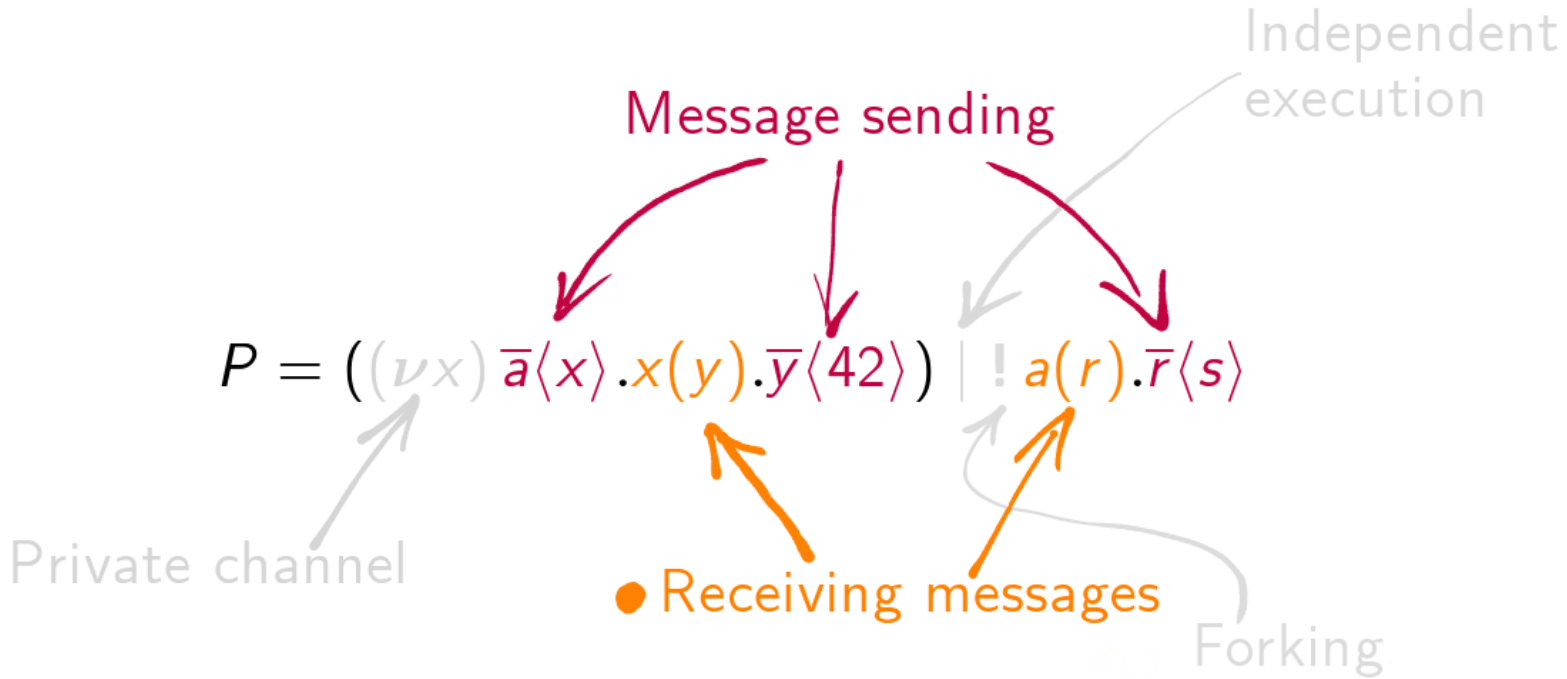

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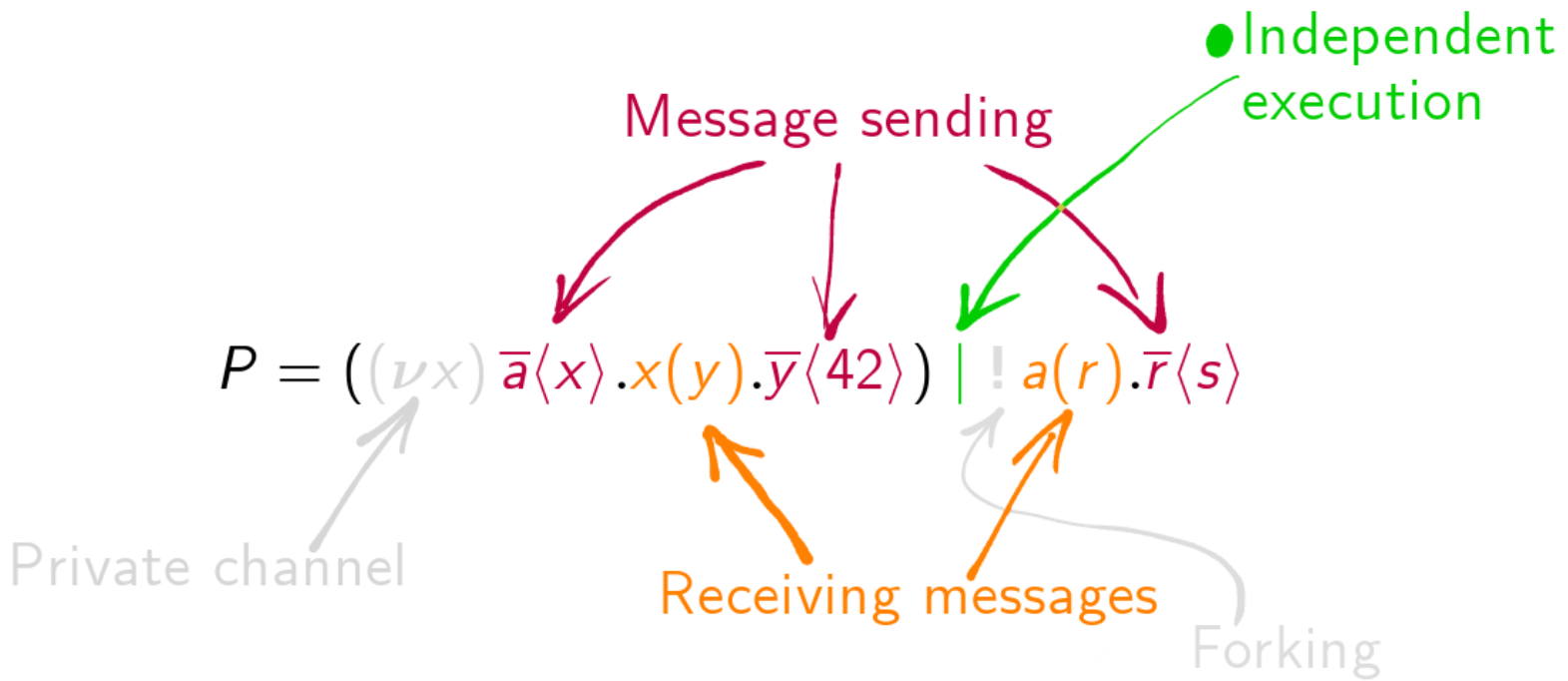

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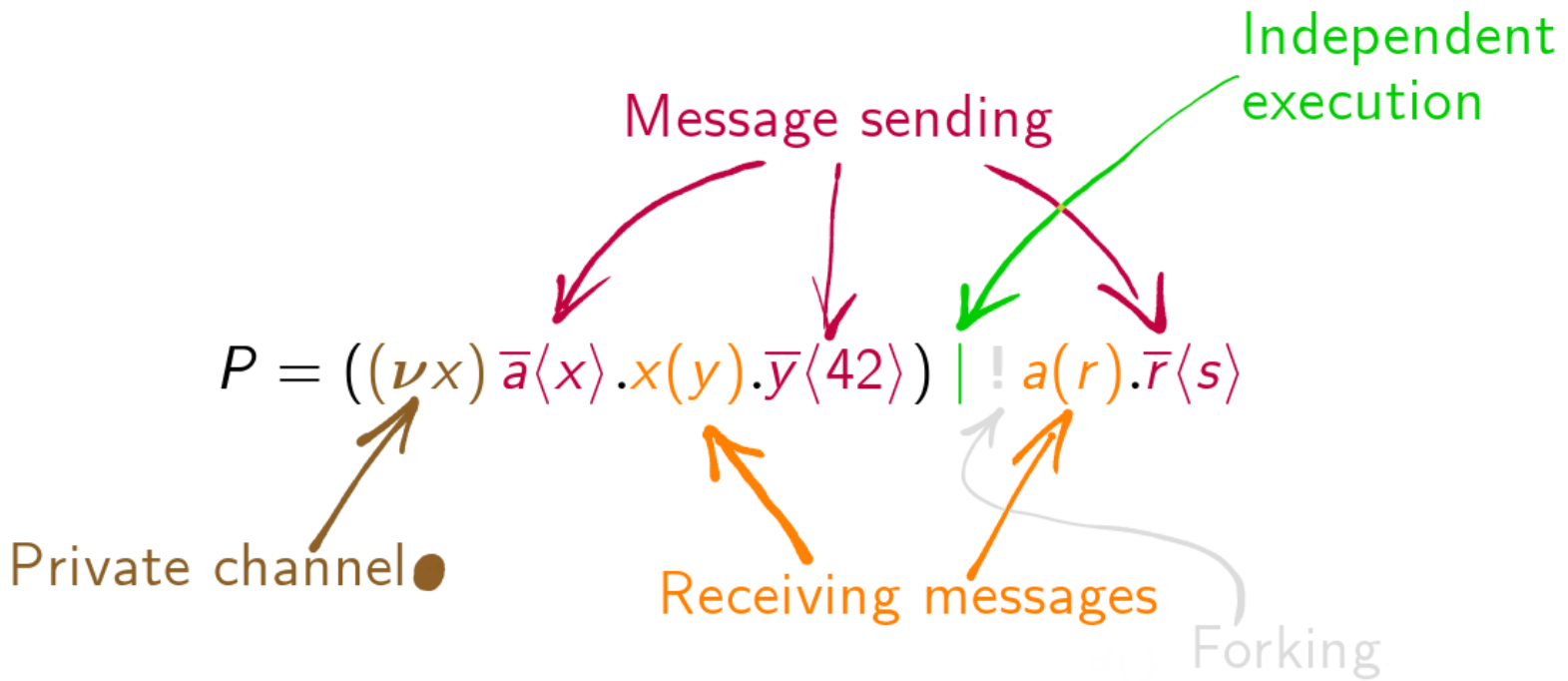

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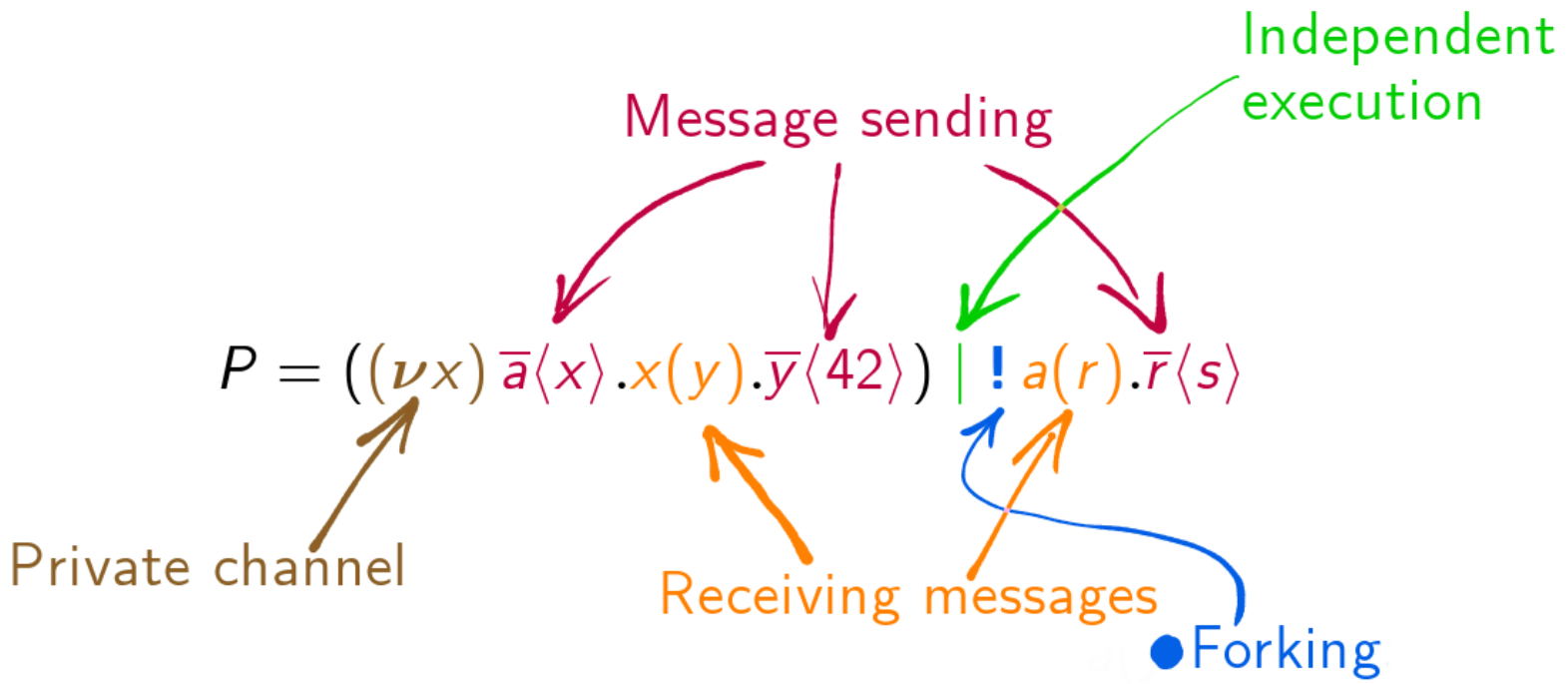

# Transitions

$$
\begin{aligned}
& ((\nu x)\, \overline{a}\langle x \rangle.x(y).\overline{y}\langle 42 \rangle) \mid a(r).\overline{r}\langle s \rangle \\
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# This Work

*Characterising* two *liveness properties* in a *mobile process* through the use of a *type system*.

# Activeness

**Definition (Activeness)**

*Activeness* $p_{\mathbf{A}}$ of a port $p \in \{a, \bar{a}\}$ in a process $P$: Ability of $P$ to *reliably* receive ($p = a$) or send ($p = \bar{a}$) a message on it.

- $\bar{s}$ active in $((\nu x)\,\bar{a}\langle x\rangle.x(y).\bar{y}\langle 42\rangle) \mid a(r).\bar{r}\langle s\rangle$,
- $\bar{s}$ not active in $((\nu x)\,\bar{a}\langle x\rangle.x(y).\bar{y}\langle 42\rangle) \mid a(r).\bar{r}\langle s\rangle \mid \bar{a}\langle w\rangle$.

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*Responsiveness* $p_{\mathbf{R}}$ of a port $p$ in a process $P$ is the ability, every time a communication occurs on that port, to continue the conversation as far as requested by the other party.

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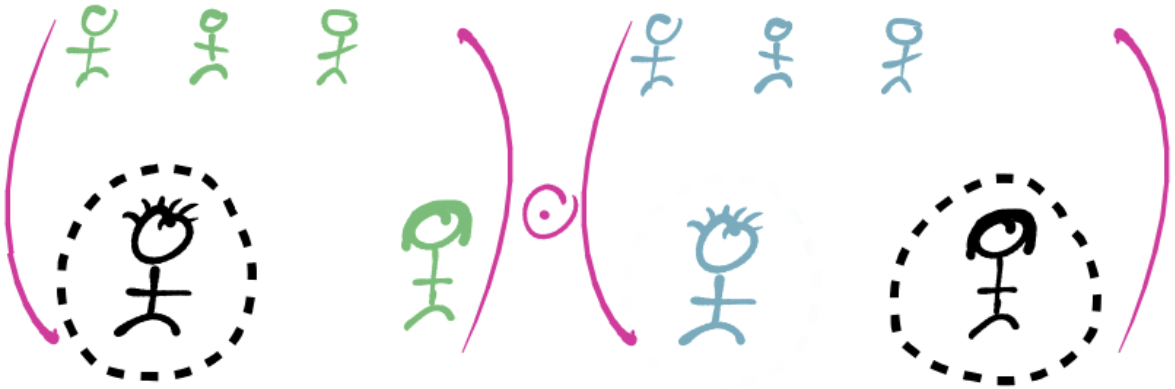
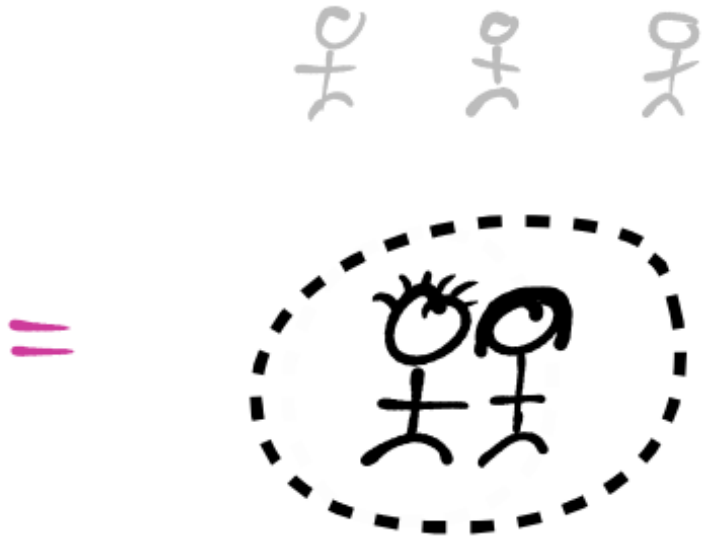


$$(L; E) \odot (L'; E') \stackrel{\text{def}}{=} (L \odot L'; (E \setminus L') \diamond (E' \setminus L))$$



# Conditional Activeness




$$(\bar{t}.a \mid \bar{u}.a) \mid (\bar{v}.\bar{a}.\bar{w}.\bar{s}) \mid (u \mid w)$$

- $\bar{s}$ depends on $v$, $a$ and $w$
- $a$ depends on any one of $t$ or $u$
- and $u$, $w$ are provided on the right
- Therefore $\bar{s}$ only depends on $v$

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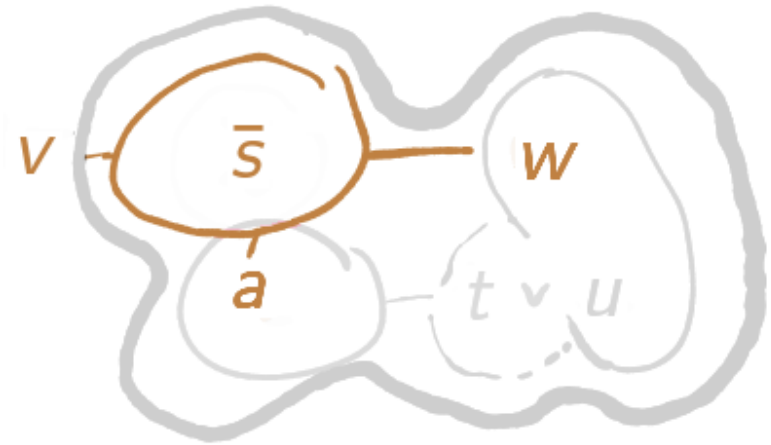


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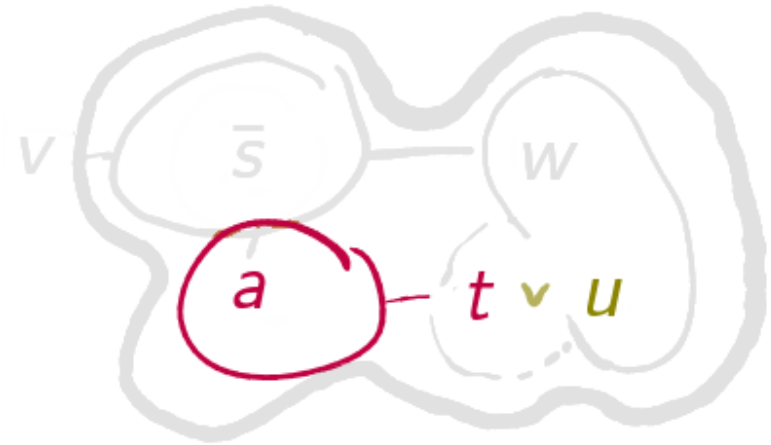


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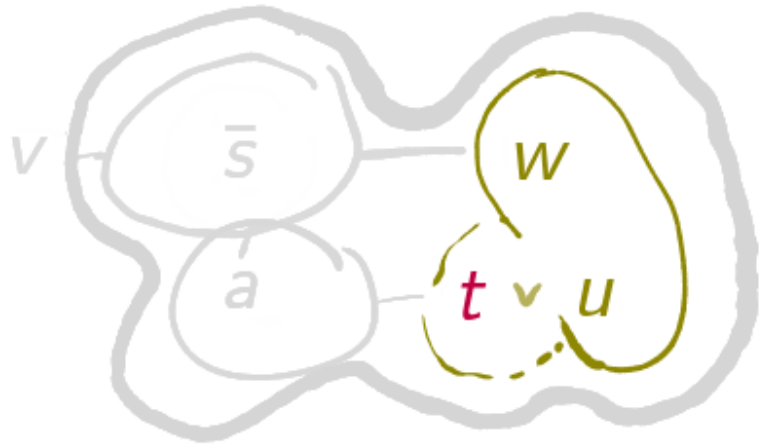


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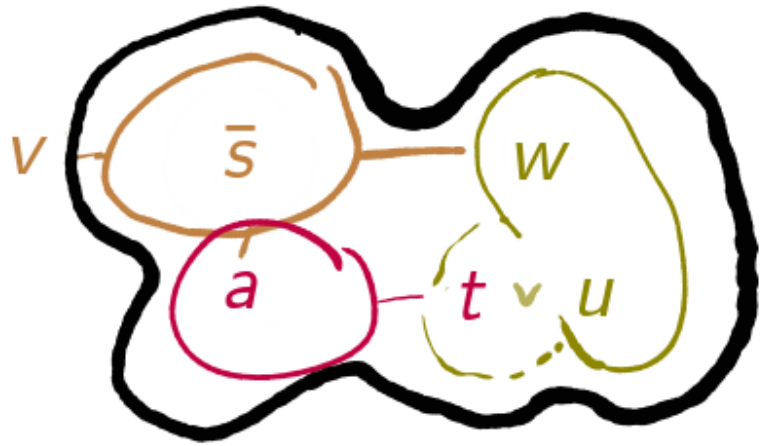


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# Labelled Dependencies

$$P = (\nu t)\left(\bar{t} \;\middle|\; t.(z|a(x).\bar{z}.\bar{x}) \;\middle|\; t.a(y).\bar{y}\right)$$

$$\bar{t}_{\mathbf{A}} \lhd \bot \quad ; \quad z_{\mathbf{A}} \lhd \bar{t}_{\mathbf{A}} \quad ; \quad a_{\mathbf{R}} \lhd z_{\mathbf{A}} \quad \Rightarrow \quad a_{\mathbf{R}} \lhd \bot$$

## Labelled Dependencies

- *Labels* $l$, $l'$, ...
- $\bar{l} \vee \varepsilon$: Only need $\varepsilon$ if "$l$" occurred.
- $l \vee \varepsilon$: Need $\varepsilon$ unless "$l$" occurred.

Set "$l$" to "*The left $t$.-prefix got consumed*".

$$\bar{t}_{\mathbf{A}} \lhd \bot \quad ; \quad z_{\mathbf{A}} \lhd l \vee \bar{t}_{\mathbf{A}} \quad ; \quad a_{\mathbf{R}} \lhd \bar{l} \vee z_{\mathbf{A}} \quad \Rightarrow \quad a_{\mathbf{R}} \lhd \top$$

# Multiplicities

$$P = ((\nu x)\, \bar{a}\langle x \rangle.x(y).\bar{y}\langle 42 \rangle) \mid !\, a(r).\bar{r}\langle s \rangle$$

- $\bar{a}^1$: One output on $a$
- $a^\omega$: Arbitrarily many inputs on $a$
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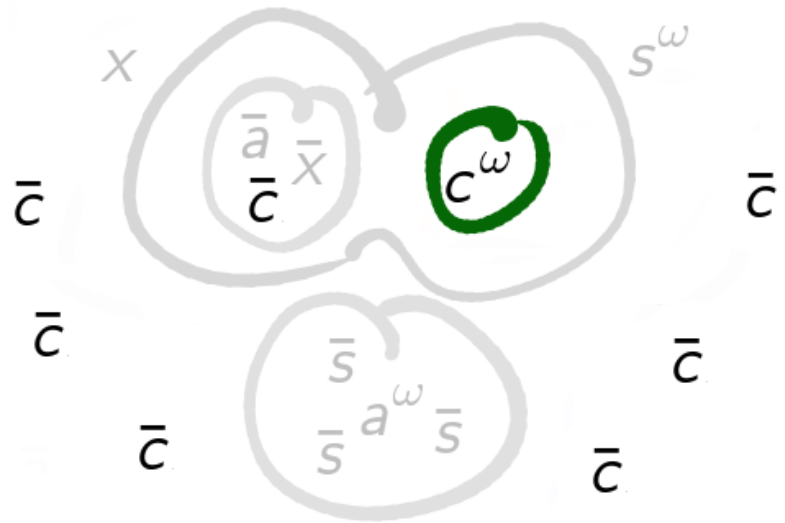


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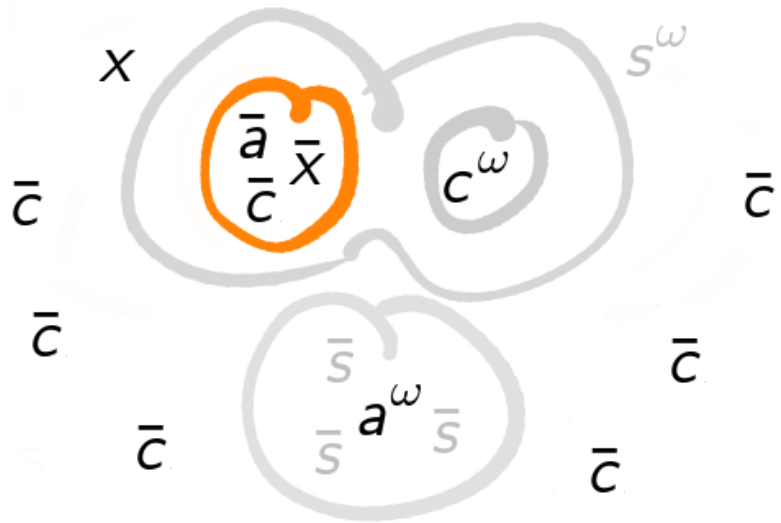


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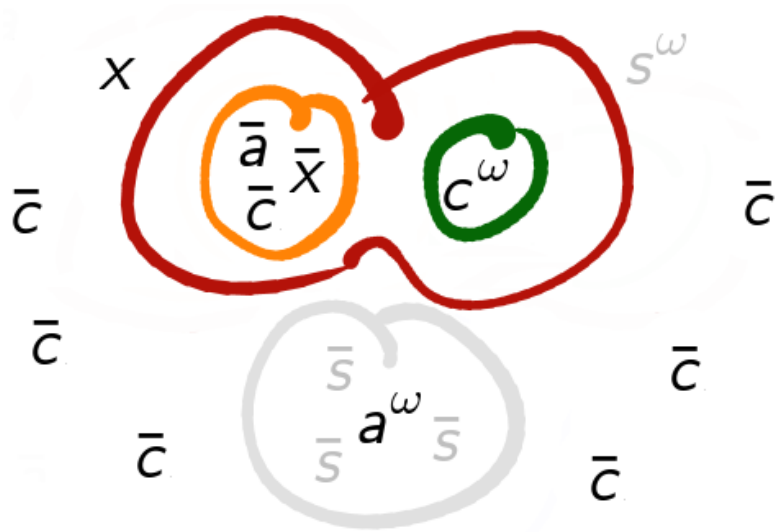


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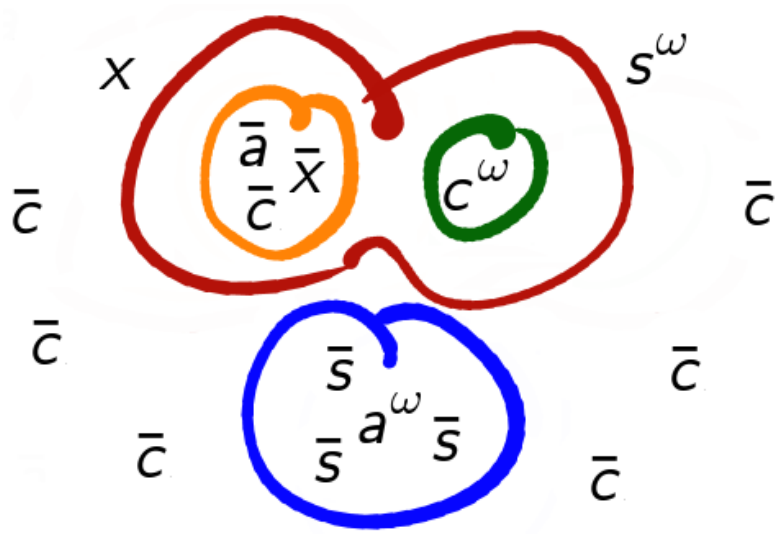


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# Conclusion

*Characterising* two *liveness properties* in a *mobile process* through the use of a *type system*.

Our contribution:

- A formalism describing liveness properties in the $\pi$-calculus
- Environment in the type $\Rightarrow$ Compositionality
- Labels $\Rightarrow$ non-transitive dependencies

More info:

- `http://maxime.gamboni.org/`